

$$v = \sqrt{2g|h| + v_0^2}.$$

7.40

This equation is very similar to the kinematics equation $v = \sqrt{v_0^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$\begin{aligned} v &= \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m}) + (5.00 \text{ m/s})^2} \\ &= 20.4 \text{ m/s.} \end{aligned}$$

7.41

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in [Falling Objects](#) that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see [Figure 7.9](#)). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.

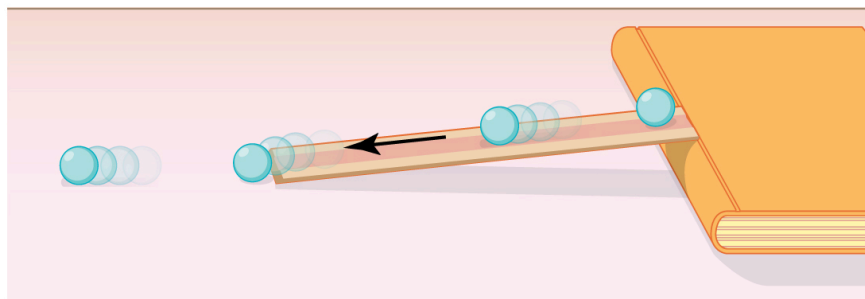


Figure 7.9 A marble rolls down a ruler, and its speed on the level surface is measured.

7.4 Conservative Forces and Potential Energy

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy

is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

Potential Energy of a Spring

First, let us obtain an expression for the potential energy stored in a spring (PE_s). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in [Elasticity: Stress and Strain](#), and states that the magnitude of force F on the spring and the resulting deformation ΔL are proportional, $F = k\Delta L$.) (See [Figure 7.10](#).) For our spring, we will replace ΔL (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude $F = kx$, where k is the spring's force constant. The force increases linearly from 0 at the start to kx in the fully stretched position. The average force is $kx/2$. Thus the work done in stretching or compressing the spring is $W_s = Fd = (\frac{kx}{2})x = \frac{1}{2}kx^2$. Alternatively, we noted in [Kinetic Energy and the Work-Energy Theorem](#) that the area under a graph of F vs. x is the work done by the force. In [Figure 7.10\(c\)](#) we see that this area is also $\frac{1}{2}kx^2$. We therefore define the **potential energy of a spring**, PE_s , to be

$$PE_s = \frac{1}{2}kx^2,$$

7.42

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance x . The potential energy of the spring PE_s does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.

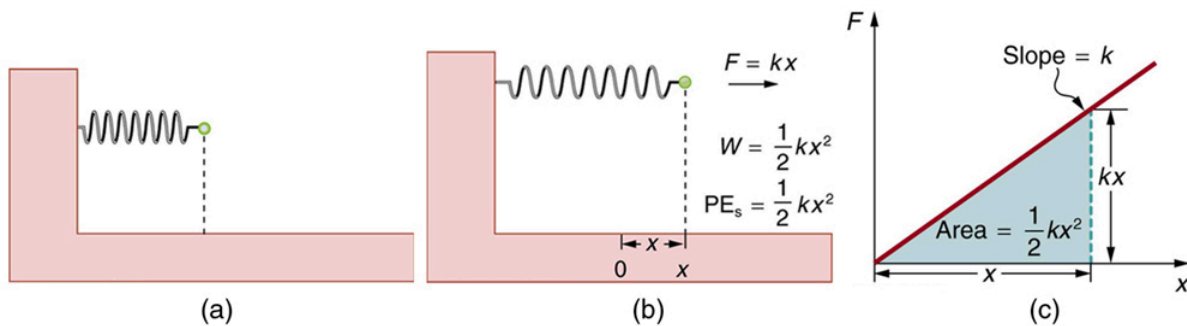


Figure 7.10 (a) An undeformed spring has no PE_s stored in it. (b) The force needed to stretch (or compress) the spring a distance x has a magnitude $F = kx$, and the work done to stretch (or compress) it is $\frac{1}{2}kx^2$. Because the force is conservative, this work is stored as potential energy (PE_s) in the spring, and it can be fully recovered. (c) A graph of F vs. x has a slope of k , and the area under the graph is $\frac{1}{2}kx^2$. Thus the work done or potential energy stored is $\frac{1}{2}kx^2$.

The equation $PE_s = \frac{1}{2}kx^2$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $PE_s = \frac{1}{2}kx^2$, where k is the force constant of the particular system and x is its deformation. Another example is seen in [Figure 7.11](#) for a guitar string.

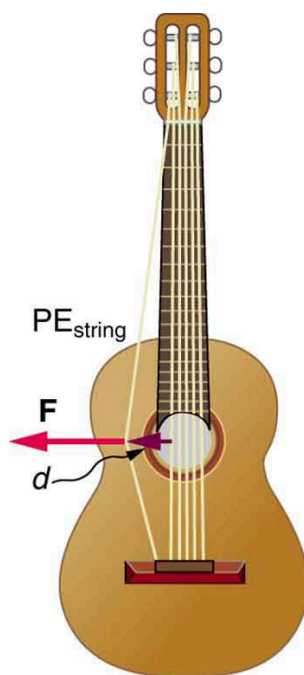


Figure 7.11 Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \Delta\text{KE}. \quad 7.43$$

If only conservative forces act, then

$$W_{\text{net}} = W_{\text{c}}, \quad 7.44$$

where W_{c} is the total work done by all conservative forces. Thus,

$$W_{\text{c}} = \Delta\text{KE}. \quad 7.45$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_{\text{c}} = -\Delta\text{PE}$. Therefore,

$$-\Delta\text{PE} = \Delta\text{KE} \quad 7.46$$

or

$$\Delta\text{KE} + \Delta\text{PE} = 0. \quad 7.47$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\left. \begin{array}{l} \text{KE} + \text{PE} = \text{constant} \\ \text{or} \\ \text{KE}_i + \text{PE}_i = \text{KE}_f + \text{PE}_f \end{array} \right\} \text{(conservative forces only)}, \quad 7.48$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, $(\text{KE} + \text{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each

force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

EXAMPLE 7.8

Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in Figure 7.12. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.

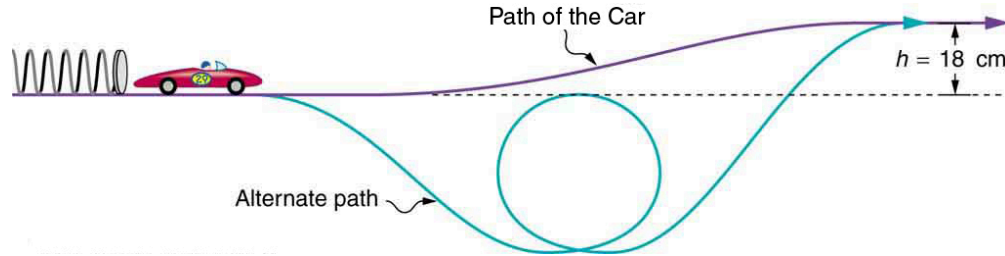


Figure 7.12 A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$KE_i + PE_i = KE_f + PE_f \quad 7.49$$

or

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2, \quad 7.50$$

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_i and h_f are zero. Furthermore, the initial speed v_i is zero and the final compression of the spring x_f is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2. \quad 7.51$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$\begin{aligned} v_f &= \sqrt{\frac{k}{m}x_i} \\ &= \sqrt{\frac{250.0 \text{ N/m}}{0.100 \text{ kg}}}(0.0400 \text{ m}) \\ &= 2.00 \text{ m/s.} \end{aligned} \quad 7.52$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f. \quad 7.53$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_f and substituting known values gives

$$\begin{aligned} v_f &= \sqrt{\frac{kx_i^2}{m} - 2gh_f} \\ &= \sqrt{\left(\frac{250.0 \text{ N/m}}{0.100 \text{ kg}}\right)(0.0400 \text{ m})^2 - 2(9.80 \text{ m/s}^2)(0.180 \text{ m})} \\ &= 0.687 \text{ m/s}. \end{aligned} \quad 7.54$$

Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in [Example 7.8](#). Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.



PHET EXPLORATIONS

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Figure 7.13



7.5 Nonconservative Forces

Nonconservative Forces and Friction

Forces are either conservative or nonconservative. Conservative forces were discussed in [Conservative Forces and Potential Energy](#). A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in [Figure 7.14](#), work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.